

Package ‘frailtyHL’

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Description Implements the h-likelihood estimation procedures for general frailty models including competing-risk models and joint models.

Depends R (>= 3.5.0), methods, Matrix, survival, cmprsk

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frailtyHL-package	<i>H-likelihood Approach for Frailty Models</i>
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Description

The frailtyHL package fits frailty models which are Cox's proportional hazards models incorporating random effects. The function implements the h-likelihood estimation procedures. For the frailty distribution lognormal and gamma are allowed. The h-likelihood uses the Laplace approximation when the numerical integration is intractable, giving a statistically efficient estimation in frailty models. (Ha, Lee and Song, 2001; Ha and Lee, 2003, 2005; Lee, Nelder and Pawitan, 2017; Ha, Jeong and Lee, 2017). This package handles various random-effect survival models such as time-dependent frailties, competing-risk frailty models, AFT random-effect models, and joint modelling of linear mixed models and frailty models. It also provides penalized variable-selection procedures (LASSO, SCAD and HL).

Details

Package:	frailtyHL
Type:	Package
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This is version 2.2 of the frailtyHL package.

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References

- Ha, I. D. and Lee, Y. (2003). Estimating frailty models via Poisson Hierarchical generalized linear models. *Journal of Computational and Graphical Statistics*, 12, 663-681.
- Ha, I. D. and Lee, Y. (2005). Comparison of hierarchical likelihood versus orthodox best linear unbiased predictor approaches for frailty models. *Biometrika*, 92, 717-723.
- Ha, I. D., Lee, Y. and Song, J. K. (2001). Hierarchical likelihood approach for frailty models. *Biometrika*, 88, 233-243.
- Ha, I. D., Jeong, J. and Lee, Y. (2017). *Statistical modelling of survival data with random effects*. Springer.
- Lee, Y., Nelder, J. A. and Pawitan, Y. (2017). *Generalised linear models with random effects: unified analysis via h-likelihood*. 2nd Edition. Chapman and Hall: London.

Examples

```
data(kidney)
kidney_g12<-frailtyHL(Surv(time,status)~sex+age+(1|id),kidney)
```

bladder

Bladder Cancer Data

Description

Bladder is an extension of Bladder0 to competing risks with 396 patients with bladder cancer from 21 centers, focusing on two competing endpoints, i.e, time to first bladder recurrence (an event of interest; Type 1 event) and time to death prior to recurrence (competing event; Type 2 event).

Usage

```
data("bladder")
```

Format

A data frame with 396 observations on the following 13 variables.

OBS Observation number

center Institution number of 24 centers

surtime Time to event

status Event indicator(1=recurrence, 2=death before recurrence, 0=no event)

CHEMO Treatment indicator representing chemotherapy(0=No, 1=Yes)

AGE Age(0, <= 65 years; 1, > 65 years)

SEX Sex(0=male, 1=female)

PRIORREC Prior recurrent rate(0, primary; 1, <= 1/yr; 2, > 1/yr)

NOTUM Number of tumors(0, single; 1, 2-7 tumors; 2, >= 8 tumors)

TUM3CM Tumor size(0, < 3cm; 1, >= cm)

TLOCC T cotegory(0=Ta, 1=T1)

CIS Carcinoma in situ (0=No, 1=Yes)

GLOCAL G graege(0=G1, 1=G2, 2=G3)

References

Sylvester, R., van der Meijden, A.P.M., Oosterlinck, W., Witjes, J., Bouffou, C., Denis, L., Newling, D.W.W. and Kurth, K. (2006). Predicting recurrence and progression in individual patients with stage Ta T1 bladder cancer using EORTC risk tables: a combined analysis of 2596 patients from seven EORTC trials. *European Urology*, 49, 466-477.

Ha, I.D., Sylvester, R., Legrand, C. and MacKenzie, G. (2011). Frailty modelling for survival data from multi-centre clinical trials. *Statistics in Medicine*, 30, 28-37.

 bladder0

Bladder cancer data

Description

Bladder0 is a subset of 410 patients from a full data set with bladder cancer from 21 centers that participated in the EORTC trial (Sylvester et al., 2006). Time to event is the duration of the disease free interval (DFI), which is defined as time from randomization to the date of the first recurrence.

Usage

```
data("bladder0")
```

Format

A data frame with 410 observations on the following 5 variables.

Center Institution number of 24 centers

Survtime Time to the first recurrence from randomization

Status Censoring indicator(1=recurrence, 0=no event)

Chemo Treatment indicator representing chemotherapy(0=No, 1=Yes)

Tustat Indicator representing prior recurrent rate(0=Primary, 1=Recurrent)

References

Sylvester, R., van der Meijden, A.P.M., Oosterlinck, W., Witjes, J., Bouffoux, C., Denis, L., Newling, D.W.W. and Kurth, K. (2006). Predicting recurrence and progression in individual patients with stage Ta T1 bladder cancer using EORTC risk tables: a combined analysis of 2596 patients from seven EORTC trials. *European Urology*, 49, 466-477.

Ha, I.D., Sylvester, R., Legrand, C. and MacKenzie, G. (2011). Frailty modelling for survival data from multi-centre clinical trials. *Statistics in Medicine*, 30, 28-37.

 cgd

Chronic Granulomatous Disease (CGD) Infection Data

Description

The CGD data set in Fleming and Harrington (1991) is from a placebo-controlled randomized trial of gamma interferon in chronic granulomatous disease. In total, 128 patients from 13 hospitals were followed for about 1 year. The number of patients per hospital ranged from 4 to 26. Each patient may experience more than one infection. The survival times (times-to-event) are the times between recurrent CGD infections on each patient (i.e. gap times). Censoring occurred at the last observation for all patients, except one, who experienced a serious infection on the date he left the study.

Usage

```
data("cgd")
```

Format

A data frame with 203 observations on the following 16 variables.

id Patient number for 128 patients

center Enrolling center number for 13 hospitals

random Date of randomization

treat Gamma-interferon treatment(rIFN-g) or placebo(Placebo)

sex Sex of each patient(male, female)

age Age of each patient at study entry, in years

height Height of each patient at study entry, in cm

weight Weight of each patient at study entry, in kg

inherit Pattern of inheritance (autosomal recessive, X-linked)

steroids Using corticosteroids at times of study entry(1=Yes, 0=No)

propylac Using prophylactic antibiotics at time of study entry(1=Yes, 0=No)

hos.cat A categorization of the hospital region into 4 groups

tstart Start of each time interval

enum Sequence number. For each patient, the infection records are in sequence number order

tstop End of each time interval

status Censoring indicator (1=uncensored, 0=censored)

References

Fleming, T. R. and Harrington, D. R. (1991). Counting processes and survival analysis. Wiley: New York.

Therneasu, T. (2012). survival: survival analysis, including penalised likelihood. <http://CRAN.Rproject.org/package=survival>. R package version 2.36-14.

CmpRsk

Model Formula of Competing Risk

Description

A CmpRsk object is used as the response variable in the model formula. It is created using the function `CmpRsk(time, index)`, where time is the event time and index is an event indicator.

Usage

```
CmpRsk(time, index)
```

Arguments

time	the event time
index	the event indicator; values of index must be sequential whole numbers where 0 denotes right censoring and positive numbers refer to different event types.

frailty.vs

Penalized Variable Selection for Frailty Models

Description

frailty.vs is variable-selection procedures (LASSO, SCAD and HL) of fixed effects in frailty models.

Usage

```
frailty.vs(formula, model, penalty, data, B = NULL, v = NULL,
alpha = NULL, tun1 = NULL, tun2 = NULL, varfixed = FALSE, varinit = 0.1)
```

Arguments

formula	A formula object, with the response on the left of a ~ operator, and the terms for the fixed and random effects on the right. e.g. formula=Surv(time,status)~x+(1 id), time : survival time, status : censoring indicator having 1 (0) for uncensored (censored) observation, x : fixed covariate, id : random effect.
model	Log-normal frailty models ("lognorm")
penalty	Penalty functions ("LASSO" or "SCAD" or "HL")
data	Dataframe used
B	Initial values of fixed effects
v	Initial values of random effects. Zeros are default
alpha	Initial value of variance of random effects.
tun1	Tuning parameter gamma for LASSO, SCAD and HL
tun2	Tuning parameter omega for HL
varfixed	Logical value: if TRUE (FALSE), the value of one or more of the variance terms for the frailties is fixed (estimated).
varinit	Starting values for frailties, the default is 0.1.

Description

frailtyHL is used to fit frailty models using h-likelihood estimation procedures. For the frailty distribution lognormal and gamma are allowed. In particular, nested (multilevel) frailty models allow survival studies for hierarchically clustered data by including two iid normal random effects. The h-likelihood uses the Laplace approximation when the numerical integration is intractable, giving a statistically efficient estimation in frailty models (Ha, Lee and Song, 2001; Ha and Lee, 2003, 2005; Lee, Nelder and Pawitan, 2017).

Usage

```
frailtyHL(formula, data, weights, subset, na.action, RandDist = "Normal",
mord = 0, dord = 1, Maxiter = 200, convergence = 10^-6, varfixed = FALSE,
varinit = c(0.163), varnonneg = FALSE)
```

Arguments

formula	A formula object, with the response on the left of a ~ operator, and the terms for the fixed and random effects on the right. e.g. formula=Surv(time,status)~x+(1 id), time : survival time, status : censoring indicator having 1 (0) for uncensored (censored) observation, x : fixed covariate, id : random effect.
data	Dataframe for formulaMain.
weights	Vector of case weights.
subset	Expression indicating which subset of the rows of data should be used in the fit. All observations are included by default.
na.action	A missing-data filter function.
RandDist	Distribution for random effect ("Normal" or "Gamma").
mord	The order of Laplace approximation to fit the mean parameters (0 or 1); default=0.
dord	The order of Laplace approximation to fit the dispersion components (1 or 2); default=1.
Maxiter	The maximum number of iterations; default=200.
convergence	Specify the convergence criterion, the default is 1e-6.
varfixed	Logical value: if TRUE (FALSE), the value of one or more of the variance terms for the frailties is fixed (estimated).
varinit	Starting values for frailties, the default is 0.1.
varnonneg	Logical value: if TRUE (FALSE), gives zero (NaN) SE for random effects when they are estimated by zeros

Details

frailtyHL package produces estimates of fixed effects and frailty parameters as well as their standard errors. Also, frailtyHL makes it possible to fit models where the frailty distribution is normal and gamma and estimate variance components when frailty structure is allowed to be shared or nested.

References

Ha, I. D. and Lee, Y. (2003). Estimating frailty models via Poisson Hierarchical generalized linear models. *Journal of Computational and Graphical Statistics*, 12, 663-681.

Ha, I. D. and Lee, Y. (2005). Comparison of hierarchical likelihood versus orthodox best linear unbiased predictor approaches for frailty models. *Biometrika*, 92, 717-723.

Ha, I. D., Lee, Y. and Song, J. K. (2001). Hierarchical likelihood approach for frailty models. *Biometrika*, 88, 233-243.

Lee, Y., Nelder, J. A. and Pawitan, Y. (2017). *Generalised linear models with random effects: unified analysis via h-likelihood*. 2nd Edition. Chapman and Hall: London.

Examples

```
#### Analysis of kidney data
data(kidney)
#### Normal frailty model using order = 0, 1 for the mean and dispersion
kidney_ln01<-frailtyHL(Surv(time,status)~sex+age+(1|id),kidney,
RandDist="Normal",mord=0,dord=1)
#### Normal frailty model using order = 1, 1 for the mean and dispersion
#kidney_ln11<-frailtyHL(Surv(time,status)~sex+age+(1|id),kidney,
#RandDist="Normal",mord=1,dord=1)
#### Gamma frailty model using order = 0, 2 for the mean and dispersion
#kidney_g02<-frailtyHL(Surv(time,status)~sex+age+(1|id),kidney,
#RandDist="Gamma",mord=0,dord=2)
#### Gamma frailty model using order = 1, 2 for the mean and dispersion
#kidney_g12<-frailtyHL(Surv(time,status)~sex+age+(1|id),kidney,
#RandDist="Gamma",mord=1,dord=2)

#### Analysis of rats data
data(rats)
#### Cox model
rat_cox<-frailtyHL(Surv(time,status)~rx+(1|litter),rats,
varfixed=TRUE,varinit=c(0))
#### Normal frailty model using order = 1, 1 for the mean and dispersion
#rat_ln11<-frailtyHL(Surv(time,status)~rx+(1|litter),rats,
#RandDist="Normal",mord=1,dord=1,varinit=c(0.9))
#### Gamma frailty model using order = 1, 2 for the mean and dispersion
#rat_g12<-frailtyHL(Surv(time,status)~rx+(1|litter),rats,
#RandDist="Gamma",mord=1,dord=2,convergence=10^-4,varinit=c(0.9))

#### Analysis of CGD data
data(cgd)
#### Multilevel normal frailty model using order = 1, 1 for the mean and dispersion
#cgd_ln11<-frailtyHL(Surv(tstop-tstart,status)~treat+(1|center)+(1|id),cgd,
#RandDist="Normal",mord=1,dord=1,convergence=10^-4,varinit=c(0.03,1.0))
```

hlike.frailty	<i>Competing Risk Frailty Models using H-Likelihood</i>
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Description

Perform hierarchical likelihood estimation of the univariate frailty model, cause-specific frailty model and subhazard frailty model. Assuming either a univariate normal or multivariate normal distribution for the random effects V , where different covariance structures can be assumed for the multivariate normal distribution.

Usage

```
hlike.frailty(formula, data, inits, order = 1, frailty.cov = "none", subHazard = FALSE,
alpha = 0.05, MAX.ITER = 100, TOL = 1e-06)
```

Arguments

formula	left-hand side is a CmpRsk object (see details), right-hand side is predictors (currently limited to numeric main effects), must include a cluster term that identifies the cluster variable.
data	dataframe containing the variables used in the formula
inits	list of initial values, three named components: beta, v and theta
order	numeric, order of the Laplace approximation, 0=no order, 1=first-order, 2=second-order; second-order only applies to models with a univariate normal distribution
frailty.cov	character string "none", "independent" or "unstructured" specifying the covariance structure for a multivariate normal distribution; "none" indicates univariate normal distribution
subHazard	logical, if TRUE fits the subhazard frailty model
alpha	numeric, 100(1-alpha) percent confidence intervals
MAX.ITER	numeric, maximum number of iterations
TOL	numeric, tolerance limit

jmfit	<i>Joint Modelling of Longitudinal and Time-to-Event Data</i>
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Description

jmfit is used to fit joint modelling of longitudinal and time-to-event data by using h-likelihood. The response of interest would involve repeated measurements over time on the same subject as well as time to an event of interest with or without competing risks.

Usage

```
jmfit(jm, data, jm2 = NULL, data2 = NULL, Maxiter)
```

Arguments

jm	list of jointmodeling objects which specify the first reponses of interest.
data	list of dataframes containing the variables used in the jm.
jm2	list of jointmodeling object which specifies the second reponses.
data2	dataframes containing the variables used in the jm2.
Maxiter	numeric, maximum number of iterations

jointmodeling	<i>Defining the Fixed and Random Models for the Mean and Dispersion Parameters in Joint Models</i>
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Description

The jointmodeling specifies jointly both the hazard model in the frailty model and the mean model in the linear mixed model.

Usage

```
jointmodeling(Model = "mean", RespDist = "gaussian", Link = NULL, LinPred = "constant",
RandDist = NULL, Offset = NULL)
```

Arguments

Model	This option specifies the mean model when Model="mean" (default).
RespDist	This option specifies the distribution of response variables (linear mixed model: "gaussian" or accelerated failure time model : "AFT" or frailty model : "FM")
Link	The link function for the linear predictor is specified by the option Link. For "AFT" or "FM" (or "gaussian") in RespDist, it is specified by "log" (or "identity").
LinPred	The option LinPred specifies the fixed and random terms for the linear predictor.
RandDist	The option RandDist specifies the distributions of the random terms represented in the option LinPred.
Offset	The option Offset can be used to specify a known component to be included in the linear predictor specified by LinPred during fitting.

kidney	<i>Kidney Infection Data</i>
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Description

The data presented by McGilchrist and Aisbett (1991) consist of times to the first and second recurrences of infection in 38 kidney patients using a portable dialysis machine. Infections can occur at the location of insertion of the catheter. The catheter is later removed if infection occurs and can be removed for other reasons, in which case the observation is censored.

Usage

```
data("kidney")
```

Format

A data frame with 76 observations on the following 10 variables.

`id` Patient number for 38 patients
`time` Time to infection since insertion of the catheter
`status` Censoring indicator(1=uncensored, 0=censored)
`age` Age of each patient, in years
`sex` Sex of each patient(1=male, 2=female)
`disease` Disease type(GN, AN, PKD, Other)
`frail` Frailty estimate from original paper
`GN` Indicator for disease type GN
`AN` Indicator for disease type AN
`PKD` Indicator for disease type PKD

References

- McGilchrist, C. A. and Aisbett, C. W. (1991). Regression with frailty in survival analysis. *Biometrics*, 47, 461-466.
- Therneasu, T. (2012). survival: survival analysis, including penalised likelihood. <http://CRAN.Rproject.org/package=survival>. R package version 2.36-14.

<code>mlmfit</code>	<i>Accelerated Failure Time (AFT) Models with Random Effects</i>
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Description

`mlmfit` is used to fit linear mixed models with censoring by using h-likelihood.

Usage

```
mlmfit(jm1, data, weights, subset, na.action, Maxiter = 200)
```

Arguments

<code>jm1</code>	This option requires <code>jointmodeling</code> object which specifies the AFT random-effect model.
<code>data</code>	dataframe containing the variables used in the <code>jm1</code>
<code>weights</code>	Vector of case weights.
<code>subset</code>	Expression indicating which subset of the rows of data should be used in the fit. All observations are included by default.
<code>na.action</code>	A missing-data filter function.
<code>Maxiter</code>	numeric, maximum number of iterations

<code>rats</code>	<i>Rats data</i>
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Description

Rats data set presented by Mantel et al. (1977) is based on a tumorigenesis study of 50 litters of female rats. For each litter, one rat was selected to receive the drug and the other two rats were placebo-treated controls. The survival time is the time to the development of tumor, measured in weeks. Death before occurrence of tumor yields a right-censored observation; 40 rats developed a tumor, leading to censoring of about 73 percent.

Usage

```
data("rats")
```

Format

A data frame with 150 observations on the following 4 variables.

<code>litter</code>	Litter number for 50 female rats
<code>rx</code>	Treatment(1=drug, 0=placebo)
<code>time</code>	Time to the development of tumor in weeks
<code>status</code>	Censoring indicator(1=uncensored, 0=censored)

References

Mantel, N., Bohidar N. R. and Ciminera, J. L. (1977). Mantel-Haenszel analyses of litter-matched time-to-response data, with modifications for recovery of interlitter information. *Cancer Research*, 37, 3863-3868.

Therneasu, T. (2012). survival: survival analysis, including penalised likelihood. <http://CRAN.Rproject.org/package=survival>. R package version 2.36-14.

 ren

Mammary tumor data

Description

The data set by presented Gail et al. (1980) is based on multiple occurrences of mammary tumors for 48 female rats. The primary outcome of interest was time to development of a mammary tumor for 23 female rats in the treatment group and 25 female rats in the control group. Initially, 76 rats were injected with a carcinogen for mammary cancer at day zero, and then all rats were given retinyl acetate to prevent cancer for 60 days. After 60 days, forty-eight rats which remained tumor-free were randomly assigned to continue being treated with retinoid prophylaxis (treatment group) or to the control group receiving no further retinoid prophylaxis. Rats were palpated for tumors twice weekly and observation ended 182 days after the initial carcinogen injection. In some cases, there were multiple tumors detected by the same day. The number of tumors ranges from 0 to 13.

Usage

```
data("ren")
```

Format

A data frame with 254 observations on the following 6 variables.

```
rat Rat id
time1 Start time
time2 Stop time
de1 Censoring indicator(1=tumor, 0=censored)
gp Treatment indicator(1=drug, 0=control)
time time2-time1 (time=time+0.01 if there are ties)
```

References

Gail, M.H. Santner, T.J. and Brown, C.C. (1980), An analysis of comparative carcinogenesis experiments based on multiple times to tumor. *Biometrics*, 36, 255-266.

Ha, I. D., Jeong, J. H. and Lee, Y. (2017). Statistical modelling of survival data with random effects: h-likelihood approach. Springer, in press.

renal	<i>Renal transplant data</i>
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Description

This is a data set from a clinical study to investigate the chronic renal allograft dysfunction in renal transplants (Ha et al., 2017). Data were available from 87 male and 25 female renal transplanted patients who survived more than 4 years after transplant. For each patient, both repeated-measure outcomes (serum creatinine levels) at several time points and a terminating event time (graft-loss time) were observed.

Usage

```
data("renal")
```

Format

A data frame with 1395 observations on the following 9 variables.

id Patient id

month Time points (month) at which the measurements of sCr were recorded

cr Serum creatinine (sCr) level

sex Sex(1=male, 0=female)

age Age(years)

icr Reciprocal of sCr(=1/sCr)

sur_time Time to graft loss

status Censoring indicator(1=graft loss, 0=no event)

first The first survival time (time to graft loss) of each patient

References

Ha, I. D., Noh, M. and Lee, Y. (2017). H-likelihood approach for joint modelling of longitudinal outcomes and time-to-event data. *Biometrical Journal*, 59, 1122–1143.

Ha, I. D., Jeong, J.-H. and Lee, Y. (2017). Statistical modelling of survival data with random effects: h-likelihood approach. Springer, in press.

test

Simulated data with clustered competing risks

Description

A data set for the cause-specific hazard frailty model assuming a bivariate normal distribution is generated using a technique similar to Beyersmann et al. (2009) and Christian et al. (2016). Let there be two event types, Types 1 and 2, as well as independent censoring. Consider a sample size $n = 100$ with $(q, n_i) = (50, 3)$. Here, q is the number of clusters and n_i is the cluster size. The random effects (log-frailties) are from bivariate normal with mean vector $(0,0)$ and variance-covariance matrix having $(1,1,-0.5)$. Data are generated from the conditional cause-specific hazard rates for each event type given the random effects. Here, for Type 1 event the two true regression parameters are $(0.6, -0.4)$ with a constant baseline hazard 2 and for Type 2 event the true parameters are $(-0.3, 0.7)$ with a constant baseline hazard 0.5, respectively. The covariates x_1 and x_2 are generated from a standard normal distribution and a Bernoulli distribution with probability 0.5, respectively. Censoring times are generated from a Uniform(0, 1.3) distribution. Under this scenario, with 25.2% censoring, the proportions of Type 1 and Type 2 events are 53.2% and 21.6%, respectively.

Usage

```
data("test")
```

Format

A data frame with 250 observations on the following 6 variables.

obs Observation number

id Id number

time Time to event

status Event indicator(2=Type 2 event, 1=Type 1 event, 0=censored)

x1 A covariate from standard normal distribution

x2 A covariate from Bernoulli normal distribution

References

Beyersmann, J., Dettenkofer, M., Bertz, H. and Schumacher, M. (2007). A competing risks analysis of bloodstream infection after stem-cell transplantation using subdistribution hazards and cause-specific hazards. *Statistics in Medicine*, 26, 5360-5369.

Christian, N. J., Ha, I. D. and Jeong, J. H. (2016). Hierarchical likelihood inference on clustered competing risks data. *Statistics in Medicine*, 35, 251-267.

Ha, I. D., Jeong, J. H. and Lee, Y. (2017). Statistical modelling of survival data with random effects: h-likelihood approach. Springer, in press.

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